

Paper Reference 9FM0/01
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

Wednesday 22 May 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

X75682A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 8 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

ADVICE

**Read each question carefully before you start to answer
it.**

Try to answer every question.

Check your answers if you have time at the end.

1. $f(z) = z^4 - 6z^3 + az^2 + bz + 145$

where a and b are real constants.

Given that $2 + 5i$ is a root of the equation $f(z) = 0$

(a) determine the other roots of the equation

$$f(z) = 0$$

(7 marks)

(b) Show all the roots of $f(z) = 0$ on a single Argand diagram.

There are blank axes on pages 34–45 in the Answer Booklet if you wish to use them.

(2 marks)

(Total for Question 1 is 9 marks)

2. The roots of the equation

$$2x^3 - 3x^2 + 12x + 7 = 0$$

are α , β and γ

Without solving the equation,

(a) write down the value of each of

$$\alpha + \beta + \gamma$$

$$\alpha\beta + \alpha\gamma + \beta\gamma$$

$$\alpha\beta\gamma$$

(1 mark)

(continued on the next page)

2. continued.

(b) Use the answers to part (a) to determine the value of

(i) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$

(ii) $(\alpha - 1)(\beta - 1)(\gamma - 1)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(7 marks)

(Total for Question 2 is 8 marks)

- 3. In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Refer to the diagram for Question 3 in the Diagram Booklet.

It shows the design for a bathing pool.

The pool, P , shown unshaded in the diagram, is surrounded by a tiled area, T , shown shaded in the diagram.

The tiled area is bounded by the edge of the pool and by a circle, C , with radius 6 metres.

The centre of the pool and the centre of the circle are the same point.

(continued on the next page)

3. continued.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a \sin 3\theta \quad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.

Given that the shortest distance between the edge of the pool and the circle C is 0.5 metres,

(a) determine the value of a
(2 marks)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T
(6 marks)

(Total for Question 3 is 8 marks)

4. The complex number $z = e^{i\theta}$, where θ is real.

(a) Show that

$$z^n + \frac{1}{z^n} \equiv 2 \cos n\theta$$

where n is a positive integer.

(2 marks)

(b) Show that

$$\cos^5 \theta = \frac{1}{16}(\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$$

(5 marks)

(c) Hence, making your reasoning clear, determine all the solutions of

$$\cos 5\theta + 5 \cos 3\theta + 12 \cos \theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(3 marks)

(Total for Question 4 is 10 marks)

5. A raindrop falls from rest from a cloud.

The velocity, $v \text{ m s}^{-1}$ vertically downwards, of the raindrop, t seconds after the raindrop starts to fall, is modelled by the differential equation

$$(t + 2) \frac{dv}{dt} + 3v = k(t + 2) - 3 \quad t \geq 0$$

where k is a positive constant.

- (a) Solve the differential equation to show that

$$v = \frac{k}{4}(t + 2) - 1 + \frac{4(2 - k)}{(t + 2)^3}$$

(5 marks)

Given that

$$v = 4 \text{ when } t = 2$$

- (b) determine, according to the model, the velocity of the raindrop 5 seconds after it starts to fall.

(3 marks)

(continued on the next page)

Turn over

5. continued.

(c) Comment on the validity of the model for very large values of t

(1 mark)

(Total for Question 5 is 9 marks)

6. Prove by induction that, for all positive integers n ,

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(4n^2-1)$$

(Total for Question 6 is 6 marks)

7. The line L_1 has equation

$$\underline{r} = \underline{i} - 2\underline{j} + 3\underline{k} + \lambda(2\underline{i} + \underline{j} - 4\underline{k})$$

and the line L_2 has equation

$$\underline{r} = 5\underline{i} + p\underline{j} - 7\underline{k} + \mu(6\underline{i} + \underline{j} + 8\underline{k})$$

where λ and μ are scalar parameters and p is a constant.

The plane Π contains L_1 and L_2

(a) Show that the vector $3\underline{i} - 10\underline{j} - \underline{k}$ is perpendicular to Π
(2 marks)

(b) Hence determine a Cartesian equation of Π
(2 marks)

(continued on the next page)

7. continued.

- (c) Hence determine the value of p
(2 marks)

Given that

- the lines L_1 and L_2 intersect at the point A
- the point B has coordinates $(12, -11, 6)$

- (d) determine, to the nearest degree, the acute angle between AB and Π
(4 marks)

(Total for Question 7 is 10 marks)

8. A scientist is studying the effect of introducing a population of type **A** bacteria into a population of type **B** bacteria.

At time t days, the number of type **A** bacteria, x , and the number of type **B** bacteria, y , are modelled by the differential equations

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 3y - 2x$$

- (a) Show that

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0$$

(3 marks)

- (b) Determine a general solution for the number of type **A** bacteria at time t days.

(4 marks)

(continued on the next page)

8. continued.

(c) Determine a general solution for the number of type **B** bacteria at time **t** days.

(2 marks)

The model predicts that, at time **T** hours, the number of bacteria in the two populations will be equal.

Given that

x = 100 and **y = 275** when **t = 0**

(d) determine the value of **T**, giving your answer to **2** decimal places.

(5 marks)

(e) Suggest a limitation of the model.

(1 mark)

(Total for Question 8 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
